AN ANALYTICAL MODEL FOR AUTOMATED DIARY MILKING

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ABSTRACT
Cow milking is without doubt one of the most important economic activities in the agricultural sector of the Nigerian economy. Understanding the characteristics of the operation of automatic milking system presents an added advantage to dairy farms. This paper presents a model of an automated milking system, with a view towards adapting it to local means of production. A closed form analytical mathematical model is derived that shows the vacuum system as a first order system.The transfer function is implemented using MATLAB. Simulink was used to simulate the modelled system using three input signals: Step input, Square Wave Pulse input and a model of the pulsator input signal - obtained from Simulink Signal builder. The output results in each case corresponded to the desired expectation.

Keywords: Claw, Cluster, Milk-line, Pulsator.

1.0. INTRODUCTION

A milking machine is a man-made mechanical system that is used to harvest milk from diary animals. Milk harvesting consists of extracting milk from the udder of the animals, transporting the milk to a storage container and storing the milk in good condition until it is picked up for processing. Milk harvesting requires a properly functioning milking machine, the cooperation of the animal and effort of the operator (Milk Tech, 2006).

An automatic milking machine may come as fully automatic; in which case, it is usually referred to as robotic milking machine. The robotic milking machine automates the whole milk harvesting process with near-zero human intervention: From when the animal is directed to the milking site, to when the milking cluster is attached to the animal, through to the disengagement of the machine from the animal, to the cleaning and sanitation of the animal udder and the machine itself and finally, to when the animal is directed out of the milking site (CowTime, 2017).

The semi-automatic milking machine is partially automated: The actual milking units and sanitation of the machine are automated while the fixing of the cluster, cleaning of the animal and directing the animal to and from the milking site is done with the help of human operator (Rossing & Hogewerf , 1997). The semi-automatic milking machine is generally referred as to Automatic Milking System (AMS) and is the focus of this paper.

In order not to affect the animal health, a properly designed milking machine should remove milk from the animal efficiently and gently. A milking machine will also not degrade milk quality from the time of removal to delivery. Milk is usually cooled during storage. In addition, it should be easy to clean and sanitize (Milk Tech, 2006).

Nigeria is dominantly an import oriented economy. With an estimated population in the excess of 160 million as at 2016 and is projected to grow to more than 392 million by 2050 (CIA, 2017). Hence, there is a stable market for dairy products. There are six major market segments in the Nigerian dairy industry: Milk, Yoghurt, Cheese, Ice Cream, Butter and Infant Formula.Currently, the milk production capacity in the country cannot satisfy the volume of demand (AgriBusinessAfrica, 2017).

Nigeria depends on importation of dairy products. Majority of cattle in the country are reared using nomadic farming and as such, most of the locally generated milk is by hand milking. To meet the current demand for dairy products, an automated system is required. This paper presents a model of an Automated Milking System, with a view towards adapting it to local means of production.

2.0. COMPONENTS OFAMS

AMS is made up of several basic component groups: The milking unit (Cluster); Vacuum production and control, Milk transportation and the cleaning and sanitation systems (DiaryNZ, 2017) . All these components are expected to work harmoniously with minimum human intervention. A diagram that depicts the integration of these various components and their interaction with each other is presented in Figure 1.

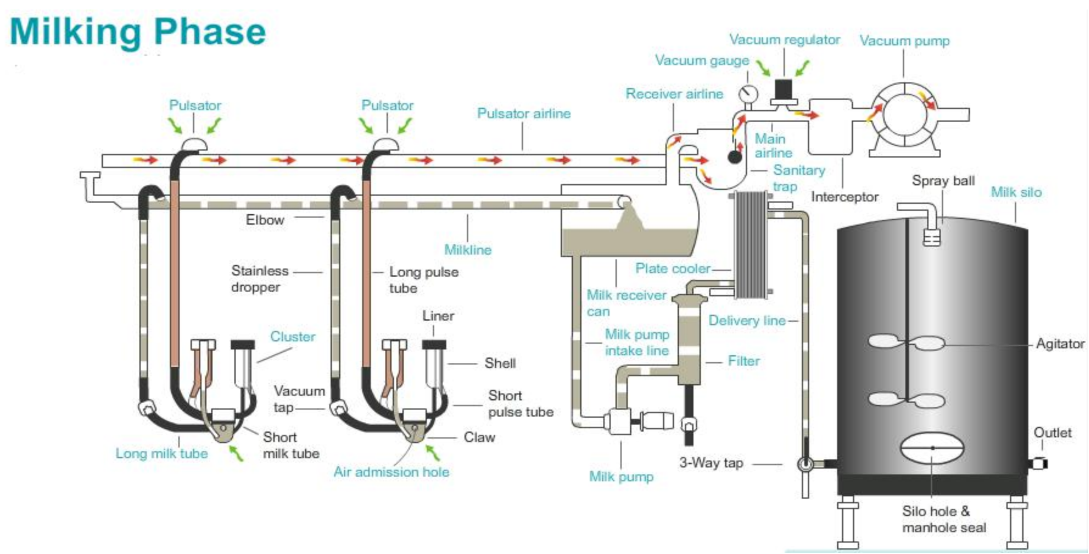

Figure 1: A Simplified Diagram of a Milking System

Source:(DiaryNZ, 2017)

A. The cluster

The milking unit is made up of several parts: The Teat-cups, the claw, The Pulsating chamber and the Connecting tubes (Milk Tech, 2006).

B. The Teat-cups

The Teat-cup consists of a soft rubber liner that is mounted in a metallic or plastic shell and is the only part of the machine that touches the udder (DiaryNZ, 2017).The space between the shell and the rubber liner is the pulsation chamber. The shell has a cavity through which a short pulse tube is connected to the claw through an air-fork. The purpose of which is to admit the vacuum within the claw into the pulsating chamber.The lower part of the Teat-cup has a short milk tube through which milk flows to the claw. The setup is analogous to suction force through the short milk tube to admit the flow of milk into the claw (CowTime, 2017) . The number of teat-cups used to milk cow is four and the number used to milk sheep is two.

An air-fork is basically a two inlet by one outlet valve that is used to alternate the pulsating chamber between the vacuum pressure in the claw and the atmospheric pressure.

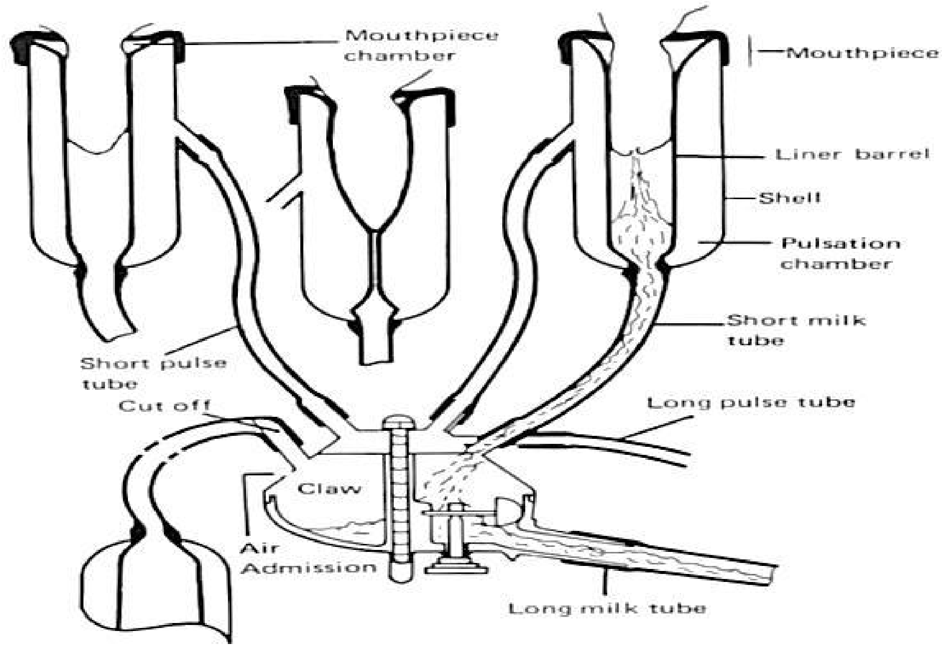

Figure 2: Diagram of a Milking Unit [1]

Source: (CowTime, 2017)

C. The Claw

The claw collects milk from all of the teat-cupsand passes same to the milk receiver. It also has a small vent for air intake to sustain continuous milk flow and maintain vacuum equilibrium pressure.The other inlet to the air fork is connected to the long pulse tube.When atmospheric pressure is passed through it, the rubber lining of the teat-cup collapsed to block the flow of milk from the udder to the claw (CowTime, 2017). A long milk tube connects the claw to the milk line and empties the milk/air content of the claw to the milk receiver.

D. The Pulsator

The arrangement of the air fork and the two pulse tubes constitute the pulsator. The switching of the air-fork between the claw vacuum and the atmospheric pressure is a very important function of the milking system. It regulates cyclic pressure changes which causes the teat cup liner to open and close around the teat-end, providing massage to the lower part of the teat (Milk Tech, 2006). The switching of pulsator can be pneumatically, mechanically or electronically controlled. Pneumatic and mechanical pulsators are now almost obsolete since they are not as reliable as electronically controlled ones (CowTime, 2017).

The principal reason for a pulsator is to limit the development of congestion and swelling in the teat tissues during milking. This can lead to cow discomfort and teat damage.Pulsation also helps to stimulate good milk let-down and to maintain a high rate of milk flow from the teat (Besier, *et al*, 2016).

The integration of the Pulsator to the teat arrangement in a cow can operate in two ways: Simultaneous Pressure (4x0) is applied to, and released from, all of the teats simultaneously, and alternating pressure (2x2) is applied to two teats while the other two are at rest and then alternated. For alternating systems, bores of 6- 8mm are typical, and for simultaneous systems bores of 8-9mm(DiaryNZ, 2017).

3.0. THE VACUUM SYSTEM

Central to the working of a Milking machine is the vacuum system. This maintains the vacuum necessary for the system's operation. The vacuum system consists of a vacuum pump, a vacuum chamber, a vacuum regulator and connecting tubes (Djordje & Miroslav , 2011).

A. The Vacuum Pump

The purpose of a vacuum pump is to extract air continuously from the milking machine system. This maintains a vacuum which allows the milking machine to operate. As was shown earlier, the pulsator alternatively works between the vacuum pressure and atmospheric pressure in order to draw-in milk from the teats. Vacuum pump performance is assessed by measuring the quantity of air flow produced.

B. The Vacuum Chamber

This is a pressurized container that has a fixed volume from which the vacuum pump is attached and continuously draws out air. The volume of the chamber is important in the determination of the system's capacitance and the configuration of the air-lines is crucial to the determination of the airflow resistance of the system.

C. Vacuum Regulators

Vacuum regulator maintains the desired level of vacuum in the AMS, despite fluctuations in air demand. It works by constantly regulating the air flow drawn from the air-lines into the vacuum chamber so the vacuum pump is continually removing the extra air that the milking machine doesn't require. This excess air is referred to as the plant's reserve air. When the vacuum of the plant suddenly falls the regulator senses this and immediately reduces the amount of reserve air it is admitting. This ensures the plant's desired vacuum level is maintained (Milk Tech, 2006).

D. The Connecting Tubes

The Connecting tubes consist of the following:

i) The air-lines through which air is extracted from the system to maintain a given vacuum level.
ii) A short pulse tube that connect shell to air fork (claw).
iii) A long pulse tube connects air fork to atmospheric pressure
iv) The milk-lines through which milk flow in the system.
v) A short milk tube that connects liner to claw.
vi) A long milk tube that connects claw to milk-line.

E. Features of pulsation systems (Pulsation cycles)

Pulsation is seen as analogous to sucking as carried out by animal infants. For an AMS pulsator, two questions may arise as to how the suction can be imitated. How long can the sucking phase and release phase be (pulsation ratio)? How often should the pulsating ratio be applied per given time (pulsation rate)? What level of pressure should be maintained during the pulsation cycles?

"Both research and field experience has shown that a relatively narrow range of pulsation rates and ratios is required to ensure good teat-end and udder health, and also to optimize milking speed.The preferred range for pulsation rate is about 55 - 65 cycles per minute and the most common pulsator ratios are 60:40 and 65:35"(DiaryNZ, 2017).

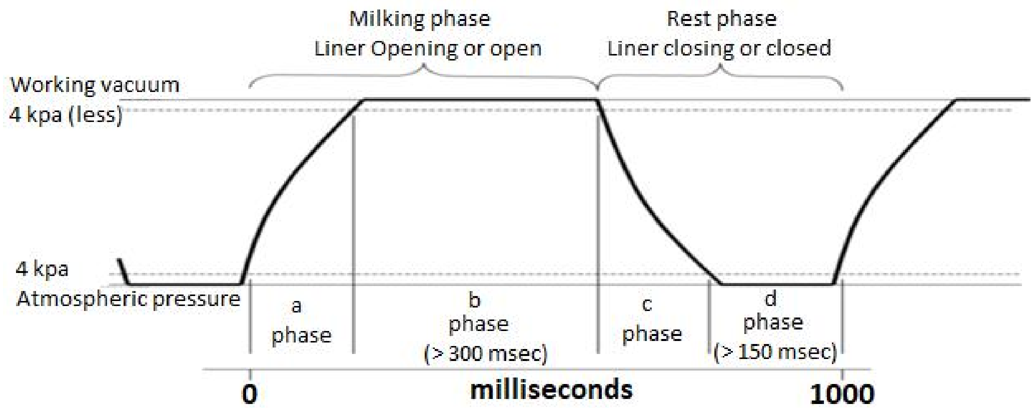

Figure 3: Pulsation Cycle

Source: (DiaryNZ, 2017)

F. Vacuum Levels

A range of vacuum pressure of 32 to 42 kPa was suggested by (ISO/DIN_5707, 2007) for gentle and efficient milking of cows. Reference (Öz, Meierhöfer, *et al*, 2010.) showed that a vacuum level of 39 kPa was found to be the system's optimum vacuum level that will minimize vacuum fluctuations as a result of some verification tests reported in (Meierhöfer, *et al*, 2011).

4.0. THE MATHEMATICAL MODEL

The Vacuum pump removes air from the vacuum chamber and creates the necessary vacuum needed for the system's operation as shown by both (Patrascioiu, *et al*, 2009) and (Stanisław, Henryk, & Mateusz, 2016). Note that, to maintain a vacuum, some amount of air is continuously allowed into the vacuum chamber, through the airlines, as the vacuum pump on the other side continuous to remove air from the chamber. Equilibrium is subsequently maintained for a constant vacuum pressure. The vacuum chamber equations are derived as shown below, using parameters in Figure 4.

Figure 4: A Simplified Diagram of a Vacuum chamber

The quantity, q, of gas that flows through a pipe is directly proportional to the pressure difference between the pipe ends.

$$q \propto \Delta p \tag{1}$$

$$q = K_1 \Delta p \tag{2}$$

Where, K_1 is the constant of proportionality and is a function of the pipe constriction and is considered to be the resistance, $R,$ to gas the flow. In other words, K_1 equals R, the gas flow resistance through the air lines, between Vacuum generator and vacuum chamber, is given by,

$$R = \frac{Change\ in\ pressure}{Change\ in\ gas\ flow\ rate} \quad (3)$$

If ΔP is a change in pressure and dq is a small change in the gas flow rate, then

$$R = \frac{d(\Delta p)}{dq} \quad (4)$$

If the pressure changes from input to output as P_i and P_o, then

$$R = \frac{P_i - P_O}{q} \quad (5)$$

In a similar manner, the change in the mass, m, of gas extracted from the vacuum chamber is directly proportional to the pressure difference at the pipe ends.

$$m \propto \Delta p \quad (6)$$

$$m = K_2 \Delta p \quad (7)$$

Where, K_2 is the constant of proportionality and is a function of the chambers storage capacity and is therefore term as the capacitance of the chamber. In other words, K_2 equals Q the capacitance of the vacuum chamber and is given as,

$$Q = \frac{Change\ in\ storaged}{Change\ in\ pressure} = \frac{qdt}{dp} = \frac{dm}{dp} \quad (8)$$

Where m=mass

$$Q = \rho \frac{dV}{dp} \quad (9)$$

Where:

ρ= gas density.

To obtain expression for$(d\rho\ )/dp$, let us start with the ideal gas law.

$$PV = nRT \quad (10)$$

Where,

P = pressure, V = volume, n = number of moles, R = molar gas constant and T = Kelvin temperature.

$$PV = \frac{R}{M}T = R_{gas}T \quad (11)$$

Where M = Molar Mass and $R/M = R\_gas = Specific\ gas\ constant.$

But $v = 1/\rho$(unit mass/specific volume =density).

$$pv = \frac{p}{\rho} = R_{gas}T \quad (12)$$

Where: p =absolute pressure, v =specific volume of gas.

Polytrophic process is a term that describes a reversible process on open or closed system of gas, which involves both heat and work transfer and is characterized by a specified combination of properties that were maintained constant throughout the process (Kirkby, 2011) .Vacuum chamber is one example of a polytrophic process.

For a polytrophic gas, the following equation is true.

$$PV^n = K = constant \tag{13}$$

Where n = polytrophic index.

But $V = m/\rho$

$$PV^n = P\left(\frac{m}{\rho}\right)^n \tag{14}$$

Therefore following the manipulation in (Ogata, 2010);

$$P\left(\frac{V}{m}\right)^n = P\frac{1}{\rho^n} = K \tag{15}$$

$$P = K\rho^n \tag{16}$$

$$\frac{dP}{d\rho} = Kn\rho^{n-1} \tag{17}$$

$$\frac{d\rho}{dP} = \frac{1}{Kn\rho^{n-1}} \tag{18}$$

But from (16)

$$\frac{P}{\rho^n} = K \tag{19}$$

Therefore, (18) becomes

$$\frac{d\rho}{dP} = \frac{\rho^n}{Pn\rho^{n-1}} \tag{20}$$

$$\frac{d\rho}{dP} = \frac{\rho}{nP} \tag{21}$$

From (12),

$$\frac{\rho}{P} = \frac{1}{R_{gas}T} \tag{22}$$

Therefore,

$$\frac{d\rho}{dP} = \frac{1}{nR_{gas}T} \tag{23}$$

Equation(9) becomes,

$$Q=V\ \frac{d\rho}{dP}=\frac{V}{nR_{gas}T} \tag{24}$$

From (5), we have

$$q=\frac{P_i-P_O}{R} \tag{25}$$

And from (8), we have

$$Qdp_o=qdt \tag{26}$$

Combining (25) and (26) gives

$$Q\frac{dp_o}{dt}=\frac{p_i-p_o}{R} \tag{27}$$

$$RQ\frac{dp_o}{dt}=p_i-p_o \tag{28}$$

Rearranging (28), gives

$$\frac{p_o}{p_i}=\frac{1}{\left(RQ\frac{d}{dt}+1\right)} \tag{29}$$

Use the Laplace Transform to produce

$$\frac{P_o(s)}{P_i(s)}=\frac{1}{(RQs+1)} \tag{30}$$

The above equation is the transfer function of a first order system, which shows that the modelled Automatic Milking System is a fist order system. Given below is a hypothetical application example, which starts by first calculating the value of Q and R.

A. Capacitance calculation

Capacitance, Q, can be calculated from (25), by substituting the values of n, V,T and R_{air}. Let us use vacuum chamber volume of 20 litres (0.02m^3) arbitrarily, and polytrophic index, n = 1, Kelvin temperature T = 300 K, and from table of specific gas constants, we obtain R_{air} = 287 Nm/kgK.

$$Q=\frac{V}{nR_{gas}T}=\frac{0.02}{1\times287\times300}=2.32\times10^{-7}m^3$$

B. Conductance Calculation

Conductance is the inverse of the resistance to the flow of gas through the piping system. It is a function of the piping dimensions and the pressure. For a typical configuration consisting of a straight tube with circular sections, having a length *(L)* much larger than the diameter (*D*), conductance *C*, is calculated by means of the following formula[31]:

$$C=\frac{135\times D^4}{L}\times P(\text{m}^3/\text{s}) \tag{31}$$

This is a case of a Laminar Flow only:

Let us arbitrarily choose the length, L, of the airline in the system to be 100 cm and the diameter of the tubing of the airliner to be 1 cm. The working pressure is approximately 40 kPa (400mbar). Put the values into (31).

$$C = \frac{135\times(0.01)^4}{1} \times 40000\ Pa = 0.054(\mathrm{m^3/s})$$

$$R = \frac{1}{C} = \frac{1}{0.054} = 18.52\ (\mathrm{s/m^3})$$

Having obtained arbitrary values for R and Q, RQ can be calculated as:

$$RQ = 4.297 \times 10^{-6}(\mathrm{s})$$

The value of RQ can be substituted into (30) as

$$\frac{P_o(s)}{P_i(s)} = \frac{1}{(4.297e{-}6s{+}1)}$$

A transfer function plot of the equation is plotted using MATLAB with a step input and the characteristics obtained are as follows. From the plot the system reached it steady state in about 0.16 millionth of a second.

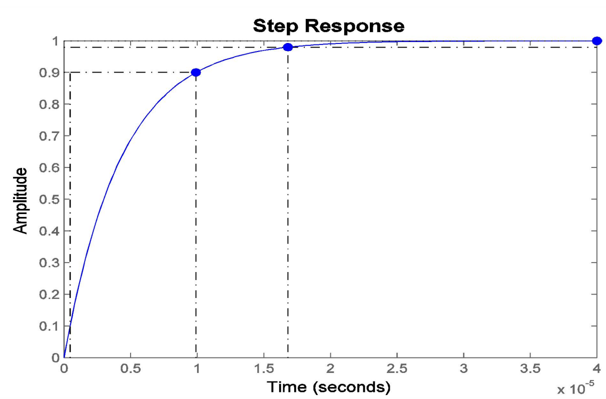

Figure 5: Aplot of the step function using Matlab

The system is next simulated using Simulink (Chaturvedi, 2010) . Three signals were used to study the response of the model. A step input, a square wave pulse input and finally a signal builder is used to develop and test a signal that represent the 60:40 ratio of the cycle of the pulsator. The outputs are as illustrated in the Figures below. Please, note that in each case the lower axes represent the input and the upper exes represents the output.

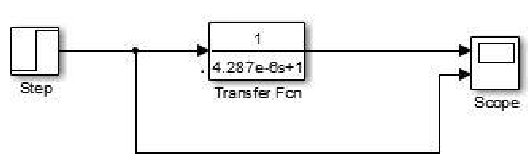

Figure 6: Model test with step input

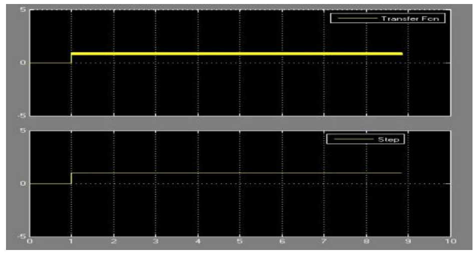

Figure 7: Shape of step input and corresponding output

Figure 6 represents the simulated system and Figure 7 shows the response. The input has the same response as the output. The model behaved as expected.

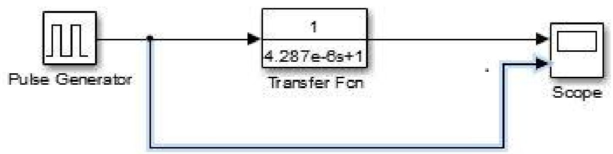

Figure 8: Square pulse input to the model

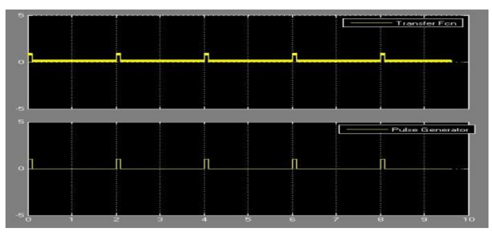

Figure 9: Square pulse input and the corresponding out

Next, the model is simulated with square pulse wave generator. Figure 9 is the simulated model and Figure 10 is the response.

The input response has the same shape as the output indicating that our model behaved as expected.

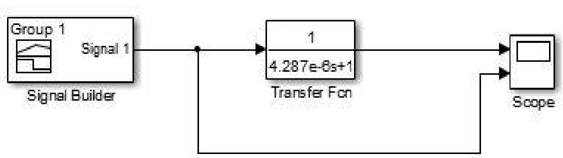

Figure 10: Shape of model input to the system

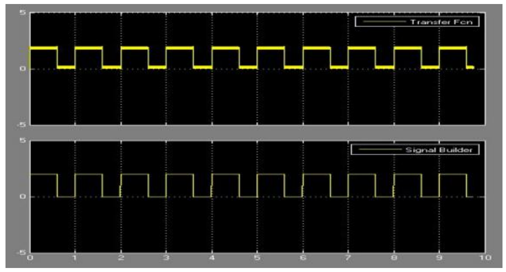

Figure 11: Output and the corresponding input of the modelled signal

Finally, Figure 10 shows a signal builder that is used to model a signal inimical to the 60:40 pulsation ratio of the pulsator. That is for a given pulsation cycle, the pulsator is open to milk flow for 60% of the time while it is close for 40% of the time. The plot is as shown in Figure 11 and the response is as expected.

5.0. ANALYSIS AND INTERPRETATION OF RESULT

Equation (30) is the transfer function of the vacuum system and it is clearly, a first order system. The dimensions of Q and that of R can be obtained from (24) and (5) respectively when we substitute for the metric units. RQ in (30) has the dimension of time and it is the transfer function's time constant.If we compute the values of R and Q, we will get the exact transfer characteristics of the vacuum system.

A. Discussion of Results

The components of AMS were identified. The function of each component was described. Although, the operating principles were well understood and explained, no attempt was made to identify the technical details required for the design and manufacturing decision of each component.

Furthermore, the operation of AMS is analyzed mathematically and a transfer function of the core system is obtained. The model obtained is an abstract one and is subject to further experimental verification to identify it practical constraints towards developing a physical system.

The results of the simulation are obtained without consideration to applied load. The applied load here would have been the mass/volume of milk extracted per specified time interval. There are literature that work extensively on this and the effect it has on animal health (Öz, *et al*, 2010.), (Meierhöfer, *et al*, 2011) and (Rossing & Hogewerf , 1997).

The project did not attempt to study the pulsator switching techniques. Pulsator design and switching is a potential area of investigation because, the pulsation ratio and the pulsation rate has direct consequences on milk throughput and on the animal health (DiaryNZ, 2017) and (CowTime, 2017).

6.0. CONCLUSION

This project identified the components of Automatic Milking System and explained the operation of an integrated machine with a view toward showing it relevance to the Nigerian economy. A mathematical model of the system was developed and it transfer function obtained as a first order system, with a view toward adapting it to local production. The model was further simulated using Simulink and the results obtained were shown to be in agreement with the required model expectations.

7.0. REFERENCES

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