PREDICTION ACCURACY OF THREE INFILTRATION EQUATIONS FOR A SANDY LOAM SOIL IN MINNA, NIGERIA.

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ABSTRACT

Prediction of soil infiltration is a complex process with many factors contributing to the rate. Three infiltration models were chosen to ascertain their predictive ability of a sandy loam soil in Minna, Nigeria. The models were Kostiakov, Philip and Horton infiltration equations. Field measurements of infiltration were conducted using the double ring infiltrometer. Parameters were developed from measured infiltration data for each of the equations and laboratory analyses of soil samples were carried out. These models were compared to determine which one most accurately predicted the measured infiltration rates from the field data. The R^2 values of the Kostiakov, Philip and Horton models were 0.979, 0.611, and 0.757, respectively. The results indicate that Kostiakov provided the best fit with the field measured data among the models tested. The root mean square error (RMSE) of the infiltration equations showed that Kostiakov had the least value of 0.723 compared to that of Philip and Horton with RMSE values of 1.669 and 2.470, respectively. All three models studied provided good overall agreement with field-measured data. However, Kostiakov provided the best fit with experimental data for the soil and is therefore recommended for the characterization of the infiltration process of the study area.

1.0 INTRODUCTION

Infiltration is the entry of water into the soil from the soil surface. Soil infiltration is an important soil quality indicator because it has important agricultural and environmental effect and is strongly affected by land management practices. The process of infiltration can go on only if there is room present for additional water at the soil surface. The available volume for additional water in the soil is affected by the porosity of the soil and the rate at which earlier infiltrated water can move away from the surface through the soil. The maximum rate at which water can enter a soil in a given condition is the infiltration capacity. If the amount of water at the soil surface is less than the infiltration capacity, all of the water available will infiltrate. If rainfall intensity at the soil surface occurs at a rate that surpasses the infiltration capacity, ponding begins and is followed by runoff over the ground surface, once depression storage is filled. Infiltration capacity which is an important soil hydrological property is influenced by soil structure, aggregate stability, particle size distribution, land use type, vegetation and topographic and climatic influences (Fu et al., 2000). Infiltration capacity is an important component in planning land disposal of waste water, selecting and setting up irrigation systems, deciding suitable water conservation techniques on agricultural lands and in the hydrological modeling of runoff processes (Criddle et al., 1956).

There are different equations for determining infiltration rates. These equations describe infiltration processes and produce different predictions for infiltration rates. They use different parameters and were developed for different purposes. In order to characterize infiltration for field applications, expression of the infiltration rate or cumulative infiltration algebraically in terms of time and certain soil parameters is important. The equations can be classified into physically based and empirical equations. Philip equation is a physically based equation. It has more advantages over the others because it does not require measured infiltration data but are based on assumptions of uniform movement of water from the surface downward and a disadvantage that the above assumption can never be fully valid.

Kostiakov and Horton equations are empirical equations. Unlike the physically based equations, they give infiltration rates based on measured infiltration data or from more approximate estimation procedures; therefore, they provide more realistic estimates when measurements can be provided for the same or very similar conditions to the site for which the prediction is to be made. Their initial parameters are determined based on actual field-measured data (Skaggs and Khaleel, 1982; Rawls et al., 1993).
However, one characteristic of infiltration that all the equations predict is an initially rapid decrease in infiltration rate with time for ponded surfaces (Skaggs and Khaleel, 1982).
Due to various works done by different researchers on infiltration rate, different infiltration equations were developed. Not all these equations will be appropriate for a particular soil type. Therefore, it becomes imperative to determine which of the equations best fits a soil type with the aid of infiltration data collected. In Nigeria, few similar studies have been carried out in this regard (Ahmed, 1982; Eze, 2000; and Idike, 2002). The aim of this study was to determine the infiltration rates of a sandy loam soil in Minna, Nigeria; predict relative infiltration rates of the soil using some time dependent infiltration equations, and to establish which of the equations studied (Philip, Horton and Kostiakov) best fits the sandy loam soil.

2.0 INFILTRATION THEORY.

2.1 PHILIP'S EQUATION.

The mathematical and physical analysis of the infiltration process developed by Phillip (1957b) separates the process into two components - that caused by a sorptivity factor and that influenced by gravity. Sorptivity is the rate at which water will be drawn into a soil in the absence of gravity; it comprises the combined effects of adsorption (the taking up by the surface of a solid or liquid of the atoms, ions, or molecules of a gas or other liquid) at surfaces of soil particles and capillarity (a phenomenon in which a liquid's surface rises, falls, or becomes distorted in shape where it is in contact with a solid) in soil pores. The gravity factor is due to the impact of pores on the flow of water through soil under the influence of gravity.

Philip (1957a) derived an equation:

$$F= \frac{1}{2} St^{-\frac{1}{2}} + A \quad (1)$$

where:

F= infiltration (rate cm/hr)

S= sorptivity (rate at which water will be drawn into the soil in the absence of gravity)

t= time (mins.)

A= gravity factor

The constants A and S can be determined by plotting the graph of F (infiltration rate) against $t^{-\frac{1}{2}}$. An advantage of the Philip equation is that it has only two parameters, which eases calculation. The two coefficients S and A can be evaluated directly from observed infiltration tests. The two parameters, however, becomes a problem when t becomes very large because it is unable to give a constant rate of infiltration. The sorptivity (S) is influenced by the initial and final moisture contents. As the moisture content approaches saturation, sorptivity tends to zero and the infiltration rate becomes equal to the field saturated hydraulic conductivity. This implies that the steady infiltration rate reached after a long time should be largely independent of the antecedent moisture content (Phillip, 1957b).

2.2 KOSTIAKOV'S EQUATION.

Kostiakov (1932) proposed the empirical equation:

$$F= Ct^{a}+ b \quad (2)$$

where:

F= infiltration rate (cm/hr)

t= time after onset of infiltration (mins)

c, a and b are constants.

The values of the constants are determined with the measured data. The first thing to be done is to plot a graph of infiltration rate, F, against time, t. Two (2) points (t_1 and F_1) and (t_2, F_2) are chosen on the graph drawn from the data. After that is done, a point $t_3 = \sqrt{t_1 t_2}$ is chosen and the corresponding F value picked as F_3. The value of b is then determined using equation (3).

$$b = \frac{F_1 F_2 - F_3^2}{F_1 + F_2 - 2F_3} \quad (3)$$

Equation (2) can be re-written as: $F–b = Ct^{a}$ (4)

Taking the logarithm of both sides of equation (4), we have

$$\text{Log}\,(F–b) = \log C + a \log t \quad (5)$$

The logarithm helps to express the equation in the form: y = Mx + c where M is the slope, x is the variable and c is the intercept along the y-axis. C and a are determined with equation (5) by substituting values.

To determine the value of F using the Kostiakov equation, the values of b, C and a are substituted into the equation (2) for each value obtained at t.

The equation for the rate of infiltration is obtained by differentiating equation (2) to obtain:

$$F= act^{a-1} \quad (6)$$

The greater the value of C, the higher the initial infiltration value (Naeth et al., 1991). The Kostiakov equation is widely and commonly used because of its simplicity, ease of finding the

two constants from measured infiltration data and reasonable fit to infiltration data for many soils over short time periods (Clemmens, 1983, Mishra et al., 2003; Haghiabi et al., 2011).
The major flaws of this equation are that it predicts that the infiltration capacity is infinite (immeasurable) at t equals zero and approaches zero for long times, while actual infiltration rates approach a steady value (Philip, 1957a; Naeth et al., 1991).
Various modifications have been made on the Kostiakov equation by different researchers like Israelson and Hanson (1967) and Mbagwu (1993),

2.3 HORTON'S EQUATION.

The Horton model of infiltration (Horton, 1939, 1940) is one of the best-known models in hydrology. Its best feature is its simplicity and good fit to experimental data. Horton recognized that infiltration rate (F) decreased with time until it approached a minimum constant rate (f_c).
The Horton equation is given as:

$$F = f_c + (f_o - f_c)\, e^{-kt} \qquad (7)$$

where

F= infiltration rate; cm/hr

f_c = the final constant infiltration rate; cm/hr

f_o = the infiltration capacity at *t* = 5; cm/hr

k = positive constant for a given soil initial condition

t = time (mins)

The parameters f_c, k and f_o must be evaluated from measured infiltration data. Subtracting f_c from both sides of equation (7) and then taking the log of each side gives the following equation for a straight line.

$$\text{Log}\,(F - f_c) = \text{Log}\,(f_o - f_c) - k\log et \qquad (8)$$

When experimental value f_c is subtracted from the experimental values of F and the logs of resulting values are plotted against time t, then k can be determined from the slope of the line (M).

$$M = k\ \text{Log}\ e$$
$$X = t\ \text{(variable)}$$
$$\text{If } M = k \log e, \text{ then } k = \frac{M}{\log e} \qquad (9)$$

Horton's equation has advantages over the Kostiakov equation. First, at t equals 0, the infiltration capacity is not infinite but takes on the finite value f_o. Also, as t approaches infinity,

the infiltration capacity approaches a nonzero constant minimum value of f_c (Horton, 1940; Hillel, 1998). Horton's equation has been widely used because it generally provides a good fit to data. Although the Horton equation is empirical in that k, f_c and f_o must be calculated from experimental data, rather than measured in the laboratory, it does reflect the laws and basic equations of soil physics (Chow et al., 1988).
However, the Horton equation is cumbersome in practice since it contains three constants that must be evaluated experimentally (Hillel, 1998). A further limitation is that it is applicable only when rainfall intensity exceeds f_c (Rawls et al., 1993). Another criticism of the Horton model is that it assumes that hydraulic conductivity is independent of the soil water content (Novotny and Olem, 1994). The Horton equation is adapted to ponded condition since it cannot be used to predict rainfall infiltration prior to surface ponding.

3.0 MATERIALS AND METHODS.

3.1 Study site

The study was carried out at the Federal University of Technology, Minna research farm, Gidan Kwano located along Minna-Bida road, Southeast of Minna, Niger State. The experimental site lies approximately on longitude of 06^0 28' E and latitude of 09^0 35' N. A detailed description of the study site is presented by Ahaneku (2014). Fig 1 shows the map of Niger State indicating the study area.

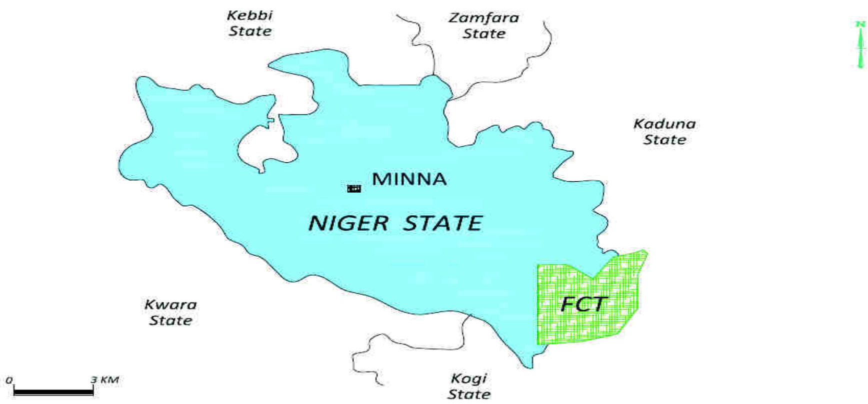

Fig 1: Map of Niger state indicating study area (Minna).

3.2 Infiltration rate measurement.

Prior to the tests, soil samples were collected from the site and taken to the laboratory for the determination of soil physical and hydraulic properties like bulk density , moisture content, porosity, hydraulic conductivity, and texture. Different points were chosen randomly (but not too far from each other), at the selected area for the infiltration runs. The infiltration measurement was carried out using double ring infiltrometers made of gauge 12 rolled iron sheet with inner and outer ring diameters of 30cm and 60cm, respectively and 25cm high. The rings were installed into the ground to a depth of 12.5cm. The infiltration tests were done at varied time interval within 2 hours twenty minutes. Infiltration runs were replicated four times.

4.0 RESULTS AND DISCUSSION

4.1 Analysis of results

The results of the physical properties of soil tested are presented in Table 1. These include: the particle size analysis, moisture content, bulk density, porosity and hydraulic conductivity. The moisture content of each replicate was the antecedent moisture content when the infiltration run was conducted.

Table 1: Some physical properties of the soil of the experimental site

Replicate	Soil textural class	%Sand	%Clay	%Silt	Moisture content (%)	Bulk density (g/cm^{3})	Porosity (%)	Hydraulic conductivity (cm/hr)
I	Sandy loam	87.112	9.00	3.888	14.18	1.60	36	2.90
II	Sandy loam	87.184	8.64	4.176	15.26	1.55	38	3.00

Table 2 shows the mean and standard deviation of the data from the infiltration runs for the observed and the infiltration equations. From the table, it can be seen that replicates III and IV have lower values of mean and standard deviation than I and II. This was as a result of the low infiltration rates during the infiltration runs of replicates III and IV which was due to higher moisture content in the soil compared to that of replicates I and II which had higher values as a result of high infiltration rates. Replicates III and IV had different moisture content because the tests were carried out several days after I and II on the same site.

Comparing the graphs of I, II, III, IV replicates in Fig 2, indicate that I and II showed a sharp decline while III and IV showed a gradual decline. This was as a result of higher moisture content and low rate of infiltration of III and IV compared to higher infiltration rates of I and II. Using the Philip's equation, it was observed that the graph of the calculated infiltration when compared to the observed in fig 3 showed a clear deviation from the graph of the observed. In contrast, the graph of cumulative infiltration (cm) against time (mins) using Kostiakov's equation when compared when compared with the observed (Fig 4) showed that the calculated data had a negligible difference due to the closeness in values of the data. Using Horton's equation, fig 5 shows a graph of infiltration rate log (F_c -F_o) against time (mins). The graph shows a high degree of deviation between the observed data and the calculated. Comparing the graphs of Philip, Horton and Kostiakov, a higher degree of accuracy was observed with the Kostiakov's equation in terms of parameters that best describes the tested soil.

Table 2: Mean and standard deviation values for the data of all equations for the infiltration runs compared with the observed data.

Replicate	I	II	III	IV
OBSERVED:				
Mean	16.105	18.978	8.811	10.516
Standard deviation	7.983	7.746	1.801	3.202
PHILIP:				
Mean	9.080	11.552	6.620	7.238
Standard deviation	3.915	4.013	1.061	1.706
HORTON:				
Mean	13.222	16.908	10.035	11.557
Standard deviation	6.769	6.660	1.110	3.002
KOSTIAKOV:				
Mean	12.006	15.260	7.901	8.805
Standard deviation	4.873	7.321	4.267	4.311

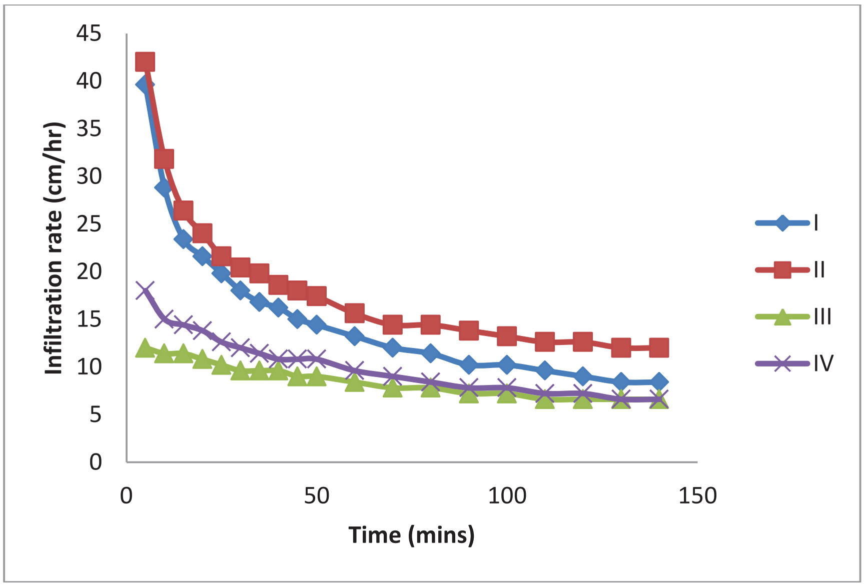

Fig 2: Graph showing the infiltration rates of all replicate runs.

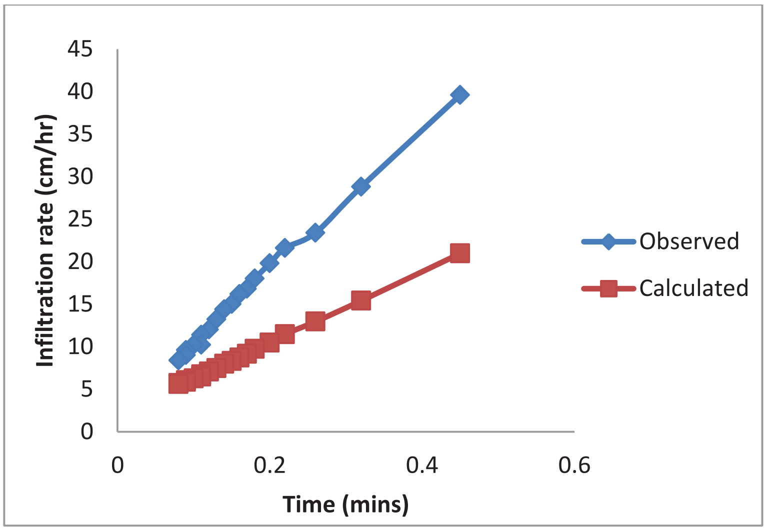

Fig 3: Graph of infiltration rate (cm/hr) against time $t^{-½}$ (mins) of observed and calculated using Philip's equation.

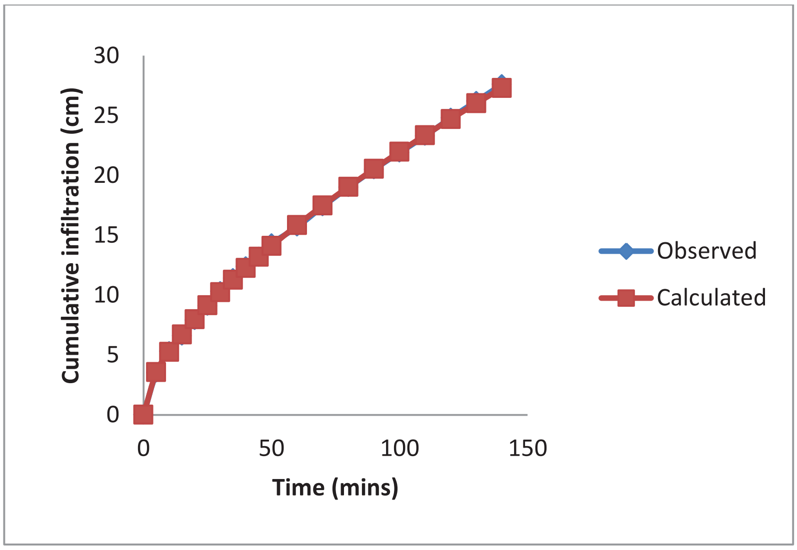

Fig 4: Graph of cumulative infiltration (cm) against time (mins) of observed and calculated using Kostiakov's equation.

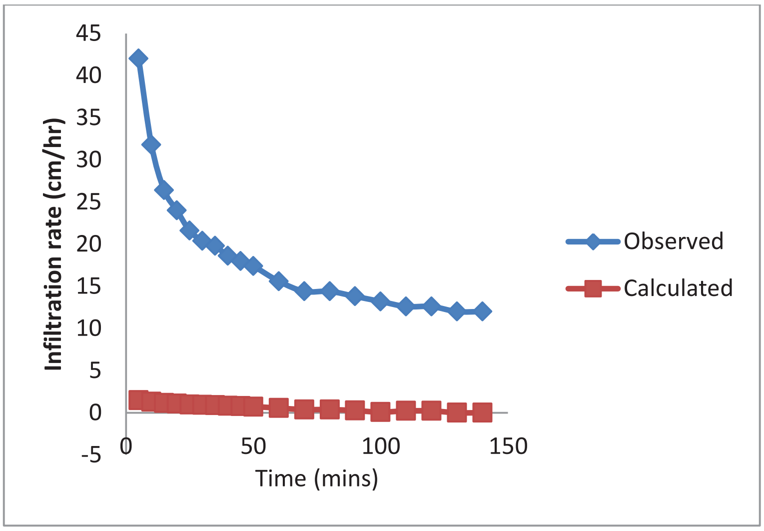

Fig 5: Graph of infiltration rate Log (F_o-F_c) (cm/hr) against time (mins) of observed and calculated using Horton's equation.

4.2 Infiltration rate prediction.

The estimated values of the models parameters are presented in Table 3. From Table 3, the average values of the time exponent of Kostiakov equation 'a' was observed to range between 0.542 and 0.662 which is in accordance with the theory of infiltration that puts the value to be positive and always less than unity. The negative values of b for the replicates III and IV were attributed to the rains before the infiltration test was carried out. The values of these parameters do not possess any specific physical meaning; however, they reflect the effect of soil physical properties of influence on infiltration as well as antecedent soil moisture content and surface conditions (Zerihun and Sanchez, 2003). The parameters of the Philip's equation were estimated by plotting a graph of infiltration rate F (cm/hr) against time $t^{-½}$ (mins) for all the replicates. A linear equation in the form of y= mx + c was gotten, the m = S and c=A for all the replicates. It can also be seen that the replicate with the highest infiltration rate (which was II) had the highest value of S, while the replicate with the lowest infiltration rate (which was III) (Fig.2) had the lowest value of S.

Table 3: Estimated values of the models parameters for each of the replicates

Replicates	Philip		Horton			Kostiakov		
	S	A	f_c	f_o	k	b	a	C
I	84.759	2.008	8.403	9.60	0.057	0.488	0.542	1.318
II	86.869	4.305	12.00	42.00	0.054	0.610	0.662	1.013
III	22.975	4.700	6.60	12.00	0.008	-0.779	0.649	0.634
IV	36.904	4.159	6.60	18.00	0.017	-0.932	0.572	0.983

4.3 Statistical analysis

Three statistical tools, namely Coefficient of determination (R^2), Root mean square error (RMSE) and Coefficient of variation (CV) were used to assess the predictive accuracy of the three infiltration equations based on field data. Typically, values of R^2 below *0.2* are considered weak, between *0.2* and *0.4*, moderate, and above *0.4*, strong. The three infiltration equations had strong values with $R^2 > 0.4$; however, Kostiakov's had the highest value (Table 4). The average R^2 value shows that Kostiakov's equation had a value of 0.979. Horton's equation comes in second with 0.757 and lastly, Philip's equation with 0.611. If one considers higher values of R^2 as indicator of goodness of fit of a model as suggested by Davidroff and Selims (1986), the results show that the Kostiakov equation provided the best fit with the experimental data for the tested soil. This result is consistent with the findings of other researchers (Ahmed 1982; Eze 2000; Idike 2002 and Ajayi et al., 2016) who tested similar models for similar soil.

Table 4: Mean statistical parameters for the infiltration equation for all replicates.

Equations	R^2	RMSE	CV
Observed	0.770	-	-
Philip	0.611	1.669	11.694
Horton	0.757	2.470	18.162
Kostiakov	0.979	0.723	3.558

Root mean square error (RMSE) with a lower value more closely predicts the measured infiltration rate, higher values provide less accurate estimates of the measured infiltration(Hartley, 1992). From the mean values of the infiltration equations for all the replicates (Table 4), Kostiakov's equation is a better fit for the tested sandy loam soil with a RMSE value of 0.723 compared with that of Philip and Horton's equation with RMSE values of 1.669 and 2.470, respectively.

The CV values for each of models tested (Table 4) shows that Kostiakov equation had the lowest value of 3.558, followed by Philip's (11.694) and lastly Horton's (18.162).

Based on the results of all the tests and analyses carried out, Kostiakov's equation was more accurate in predicting the infiltration rate of the tested soil than the other models.

5.0 Conclusions.

The infiltration rates of a sandy loam soil in Minna, Nigeria were determined in the field. Time – dependent infiltration models were used to compare the observed data. Results indicated that Kostiakov equation best predicted the infiltration rates of the soil compared with the other models. Kostiakov equation had an average R^2 value of 97.9% compared with those of Horton and Philip which had average R^2 values of 75.7% and 61.1%, respectively.

The results further showed that antecedent soil moisture content affected infiltration rates.

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