MODELLING CHANNEL ROUGHNESS COEFFICIENT USING DIMENSIONAL ANALYSIS

M.Y. Kasali[1] and A.O. Ogunlela[2]
[1]National Centre for Agricultural Mechanization, Ilorin, Kwara State, Nigeria.
E- mail: yommekasali@yahoo.com
[2] Department of Agricultural and Biosystems Engineering
University of Ilorin, Ilorin, Kwara State, Nigeria
E- mail: aogunlela@yahoo.com

ABSTRACT

A mathematical model for predicting the roughness coefficient of a channel was developed using dimensional analysis based on the Buckingham's π theorem. Dimensional analysis was used to establish the relationship between the fluid, flow and channel geometric properties. This relationship was subjected to a multiple regression analysis using the field data obtained from the channels, leading to the prediction of the roughness coefficient. The predicted roughness coefficient was validated with Manning's roughness equation and a high coefficient of correlation of 0.90 and a coefficient of determination, R^2 of 0.814 between the predicted and Manning's coefficients were obtained. The analysis of variance (ANOVA) showed that there was no significant difference between them at 5% level of significance.

KEYWORDS: Roughness coefficient, manning's equation, dimensional analysis, Buckingham's π theorem, dimensionless terms.

1. INTRODUCTION

The roughness coefficient of a channel indicates the degree of the roughness of a channel surface. It defines the extent of the resistance of the surface to flow. The roughness characteristics of an open channel are widely dependent on the geometric, hydraulic and the flow parameters of the channel.
This implies that the variation of roughness coefficient occurs due to many contributing factors such as surface, vegetation, channel irregularity, channel alignment, silting and scouring, obstruction as well as discharge of the channel. The determination of the roughness coefficient presents a provocative and a creative task of the contemporary hydraulics of open channel flows (Zic *et al*, 2009). Its determination is an interdisciplinary task because according to Zic *et al*. (2009), it includes the knowledge of hydrology, statistics, hydromechanics, hydraulics, geology and mechanics.

The value of n varies in space and time. It varies not only along the bed but also with the changes in water level. According to Abdul Ghaffar *et al*. (2004), most of the hydraulic computations related to indirect estimates of discharge require an evaluation of the roughness characteristics of the channel. Channel morphology depends on the interaction between fluid flow and the erodible materials in the channel boundary (Abdul-Ghafar *et al*, 2004).

In general, effective factors in estimation of roughness coefficients include the roughness of channel bed materials and the cross section shape of the channel (Ebrahimi *et al*, 2008). Since the last century, many researchers in the developed countries have been developing different empirical equations for the determination of the roughness coefficient in order to adapt this to the natural morphology of the natural rivers in their respective countries.

These researchers have used a number of flow resistance equations involving grain roughness, form roughness and a combination of both, but the Manning's equation has been widely used internationally for predicting roughness values in channels (Abdul Gafar *et al*, 2004).

One of the key steps in the process of mathematical modeling is to determine the relationship among the channel flow variables. This could be carried out through dimensional analysis; which is a method for helping determine how variables are related and for simplifying a mathematical model. It is based on two assumptions that:

1. Physical quantities have dimensions (Mass, M; Length, L; and Time, T)
2. Physical laws are unaltered when changing the units measuring the dimensions.

It is a useful tool for developing prediction equation of various physical systems. The methodology of dimensional analysis reduces the physical quantities pertinent to a system to dimensionless groups (Degirmencioghi and Srivastava, 1996). It employs the Buckingham's π- theorem to develop the relationship between the materials bed's roughness, the water flow and the channel's geometric parameters. The theorem states that:
'If there are n variables (Independent and dependent variables) in a physical phenomenon and if these variables contain m fundamental dimensions (M, L, T), then the variables are arranged into (n-m) dimensionless terms. Each term is called π-term'.
The equation is in the form:

$$\Phi(\pi_1, \pi_2, \pi_3, \ldots\ldots\ldots\ldots \pi_m) = 0 \tag{1}$$

Dimensional analysis is a mathematical technique which allows the enlightened design of experimental investigation. It assists in experimental investigation by reducing the number of variables in the problem. The result of the analysis is to replace an unknown relation between n variables by a relationship between a smaller, n-m, of dimensionless groups. Any reduction in the number of variables greatly reduces the labour of experimental investigation.

Many empirical formulae have been developed to estimate the values of roughness based on particle size distribution curve of surface bed material (French, 1985; Nguyen and Fenton, 2004). However, these formulae are often generated for natural rivers or in the laboratory set- up where all conditions are quite different from the field and not for constructed canals.

The objective of this study, therefore, was to develop a roughness coefficient that would incorporate and relate the geometric and hydraulic properties of a channel with flow and fluid properties using dimensional analysis.

2. MATERIAL AND METHODS

2.1 Experimental Site

The experiment was carried out at the National Centre for Agricultural Mechanization (NCAM), Ilorin. Ilorin is geographically located in the middle belt of Nigeria with a vegetation of derived savannah, and is situated on a longitude of 4^0 30' E and latitude of 8^0 26' N. It receives an average of 1200 mm annual rainfall. The soil of the experimental site is sandy loam and contains 12.48% clay, 18% silt and 69.52% sand. It is classified as Hyplustalf of Eruwa and Odo-owa series, developed from the parent materials consisting of micaceous schist and gneiss of basement complex which are rich in Ferro-magnesium materials (Ahaneku and Sangodoyin, 2003).

2.2 Experimental Design

Five different canal lining materials were examined. The five treatments were:
Treatment I - Concrete (1:2:4)
Treatment II – Clay- Cement (6:1)
Treatment III - Burnt Cementitious Clay
Treatment IV – Clay Soil
Treatment V - Termite Mound.

Forty five (45) channels of these treatments were dug and laid in a completely randomized design, in a 5^3 x 3 factorial layout. The five treatments were replicated three times, using different levels of slopes for each material. The levels of slopes were 2%, 5% and 7%, respectively. Each channel was 15 m long, a length enough for the observation of all the flowing parameters, with side slopes of 2:1.

2.3 Experimental Procedure

Water was discharged into the Channels corresponding to heads of 2 cm, 3 cm, 6 cm and 8 cm, respectively. Measurements of canal cross-section, including width of cross-section were taken for each of the canal and an automatic flow meter was used to determine the flow velocities at these heads. The wetted area, wetted perimeters and the hydraulic radii were determined at the various cross- sections. A relationship was obtained by dimensional analysis based on the Buckingham's π theorem. The relationship obtained was subjected to regression analysis using the parameters/ data from the experimental set-up, viz: velocity (V), area (A), Depth of flow (d), hydraulic radius (R), discharge (Q) and slope (S). These data were subjected to multiple regression analysis.

2.4 Model Development

The major condition for the development of this model was that the flow in the channel was uniform. In the development of the equation, the roughness coefficient of the channel was considered to be a function of the hydraulic radius of the channel(R), the slope (S), the channel discharge, Q; flow depth, d; and acceleration due to gravity, g. This is expressed mathematically as in Equation 2. Table 1 is the summary of the variables and their dimensions.

$$n = f(R, Q, S, \mu, g, d, v) \quad (2)$$

or

$$f(n, R, Q, S, \mu, g, d, v) = 0 \quad (3)$$

The total number of variables, N is eight, while the number of fundamental dimension (m) is three, hence the number of π - terms is N-m i.e 8-3=5.
It follows that the number of π - terms in the equation can be expressed as:

$$f(\pi_1, \pi_{2,} \pi_3 \pi_4, \pi_5) = 0 \quad (4)$$

Therefore:

$$\pi_1 = R^a \cdot \mu^b \cdot V^c . d \quad (5)$$

$$\pi_2 = R^a \cdot \mu^b \cdot V^c . Q \quad (6)$$

$$\pi_3 = R^a \cdot \mu^b \cdot V^c . S \quad (7)$$

$$\pi_4 = R^a \cdot \mu^b \cdot V^c . n \quad (8)$$

$$\pi_5 = R^a \cdot \mu^b \cdot V^c . g \quad (9)$$

Where π_1 to π_5 are dimensionless terms, while a, b, and c are exponents to be determined by dimensional analysis.

Table 1 Dimensions Influencing Roughness Coefficient

Physical variables	Symbols	Dimensions
Hydraulic radius	R	L
Slope	S	L/L
Discharge	Q	L^3T^{-1}
Viscosity	μ	$ML^{-1}T^{-1}$
Acceleration due to gravity	G	LT^{-2}

Flow depth	D	L
Velocity	V	LT^{-1}

***π_1* - Term**

Replacing the right hand side of Equation 5 with the corresponding dimensions of the variables and the dimensionless term on the left hand side in the $M^0L^0T^0$, Equation 10 was obtained as follows:

$$M^0L^0T^0 = L^o\,(ML^{-1}T^{-1})^b\ (LT^{-1})\ (L \tag{10}$$

Equating the exponents of μ, L and T on both sides, we have
Power of M, b = 0; Power of L, a – b + 1 = 0; Power of T, b – c = 0
By solving the above equation simultaneously, we have a = -1, b = 0, c= 0
Substituting the above values in equation (10) resulted to:

$$\pi_1 = \frac{d}{R} \tag{11}$$

Repeating the above procedure for the remaining dimensionless terms, Equations 12 to 15 were obtained:

$$\pi_2 = \frac{Q}{R^2 V} \tag{12}$$

$$\pi_3 = S \tag{13}$$

$$\pi_4 = n \tag{14}$$

$$\pi_5 = \frac{R}{V^2}.g \tag{15}$$

Substituting the dimensionless terms $\pi_1, \pi_2, \pi_3, \pi_4,$ and π_5 into Equation 4 yields the following expression:

$$f(n, \frac{d}{R}, \frac{Q}{R^2V}, S, \frac{R}{V^2}.g) = 0 \tag{16}$$

Equation 16 shows the relationship between the various parameters and the different dimensionless combinations obtained. This expression does not give the exact relationship between these parameters. The relationship could only be determined through values from the experimental field data.
Equation 4 can be written as a function of any of the others as in Equation 17:

$$\pi_4 = f(\pi_1, \pi_2, \pi_3, \pi_5) \tag{17}$$

$$n = f(\frac{d}{R}, \frac{Q}{R^2V}, S, \frac{R}{V^2}.g) \tag{18}$$

The dimensionless terms in Equation 17 are combined as in Equation 19.

$$\pi_4 = f(\pi_{13}, \pi_{25}) \tag{19}$$

The numbers of π terms are reduced by division (Equations 20 and 21) according to Shefii *et al*. (1996) as follows:

$$\pi_{13} = \pi_1^{-1} x \pi_3 = \frac{RS}{d} \tag{20}$$

$$\pi_{25} = \pi_2^{-1} x \pi_5 = \frac{R^2 V}{Q} x \frac{Rg}{V^2} = \frac{R^3 g}{QV} \qquad (21)$$

$$\pi_4 = f\left(\frac{RS}{d}, \frac{R^3 g}{QV}\right) \qquad (22)$$

Therefore;

$$n = f\left(\frac{RS}{d}, \frac{R^3 g}{QV}\right) \qquad (23)$$

$$n = K\left[\frac{R^3 g}{QV}\right]^a \left[\frac{RS}{d}\right]^b \qquad (24)$$

The exponents of Equation 24 were determined by regression analysis using the experimental field data. The equation was first log - transformed to yield a linear expression for easy determination of the components as follows:

$$Y = \ln K + a x_1 + b x_2 \qquad (25)$$

$$\ln n = \ln K + a \ln\left(\frac{R^3 g}{QV}\right) + b \ln\left(\frac{RS}{d}\right) \qquad (26)$$

where:

$$Y = \ln n$$

$$X_1 = \ln\left(\frac{R^3 g}{QV}\right)$$

$$X_2 = \ln\left(\frac{RS}{d}\right)$$

Using data from the channel flow experiment, the values of a and b in Equation 24 were obtained by regression analysis using the SPSS 16.0 statistical package. The model coefficients are in Table 2, while the summary and the analysis of variance for the model are in Tables 3 and 4, respectively.
The resulting regression equation is as follows:

$$\ln n = -0.94 + 0.36 \ln\left(\frac{R^3 g}{QV}\right) + 0.39 \ln\left(\frac{RS}{d}\right) \qquad (27)$$

From Table 2, ln K= -0.94; a = 0.36 and b= 0.39
Ln K= -0.94
Therefore, K = $e^{-0.94}$ = 0.391

Coefficients a, b, and constant K from the regression equation were substituted in Equation 27 as exponents in Equation 28 as follows:

$$n = e - 0.94\left[\frac{R^3 g}{QV}\right]^{0.36}\left[\frac{RS}{d}\right]^{0.39} \quad (28)$$

$$n = R^{3(0.36)} g^{0..36} Q^{-0.36} V^{-0.36} R^{0.39} S^{0.39} d^{-0.39} 0.391 \quad (29)$$

$$n = R^{1.08}(2.275) Q^{-0.36} V^{-0.36} R^{0.39} S^{-0.39} d^{0.39} 0.391 \quad (30)$$

$$n = (2.275x0.391) R^{1.47} Q^{-0.36} V^{-0.36} R^{-0.39} S^{-0.39} d^{0.39} \quad (31)$$

$$n = 0.89 R^{1.47} Q^{-0.36} V^{-0.36} S^{0.39} d^{-0.39} \quad (32)$$

$$n = 0.89\left[\frac{R^{1.47} S^{0.39}}{Q^{0.36} V^{0.36} d^{0.39}}\right] \quad (33)$$

Equation 33 can be used to predicte roughness coefficient.

2.5 Model Validation

The equation was validated using Manning's roughness equation:

$$n = \frac{R^{2/3} S^{1/2}}{V} \quad (34)$$

The relationship between the predicted equation and the Manning's equation is presented in Figure 1. The Manning's and predicted roughness coefficients were compared as in Table 5 using the least significant difference (LSD); at 1% and 5% level of significance to ascertain whether there were significant differences between them.

Table 2: Model Coefficients

	Unstandardized Coefficients		Standardized Coefficients		
	B	Std. Error	Beta	T	Sig.
Constant (ln K)	-0.94	0.086		-10.929	0.001
Ln R^3g/QV	0.36	0.015	0.879	23.759	0.001
Ln RS/d	0.39	0.011	1.280	34.607	0.001

Table 3: Model Summary

R	R Square	Adjusted R Square	Std. Error of the Estimate
0.933	0.871	0.870	0.062

Table 4: Analysis of Variance of the Model

	Sum of Square	Df	Mean Square	F	Sig.
Regression	4.578	2	2.289	598.83	0.001
Residual	0.677	177	0.004		
Total	5.255	179			

Table 5: Analysis of Variance between Manning's and predicted Roughness Coefficient

	Sum of Squares	Df	Mean Square	F
Between Groups	2.7778E-07	1	2.7778E-07	0.072598ns
Within Groups	3.3671E-04	88	3.8263E-06	
Total	3.3699E-04	89		

ns: No significant different at $P \leq 0.05$

3. RESULTS AND DISCUSSION

The regression analysis showed that there was high correlation between the dependent and independent variables. The relationship between the dependent variable and the independent variables is as in Figure 2. The coefficient of correlation from the regression analysis is 0.93 (Coefficient of Determination, R^2= 0.871). This shows that there was a very high correlation between the dependent and the independent variables. This implied that 87.1% relationship was found among the independent variables.

The adjusted R^2 showed that 87% of the variables can explain the model. Generally, the higher the value of the adjusted R^2, the more significant the model. It is therefore, explicit to say that the variables can adequately explain the model.

The validation of the predicted equation as shown in Figure 1 gave a very high correlation of 0.90 (Coefficient of Determination, R^2 = 0.814). It also revealed that there was a good agreement between the predicted equation and Manning's equation.

The Manning's and predicted roughness coefficients were compared in Table 5, using the least significant difference (LSD); at 1% and 5% level of significance; there were no statistical differences between them. Therefore, these results show that the interaction between the parameters of the channel has successfully been modelled using dimensional analysis.

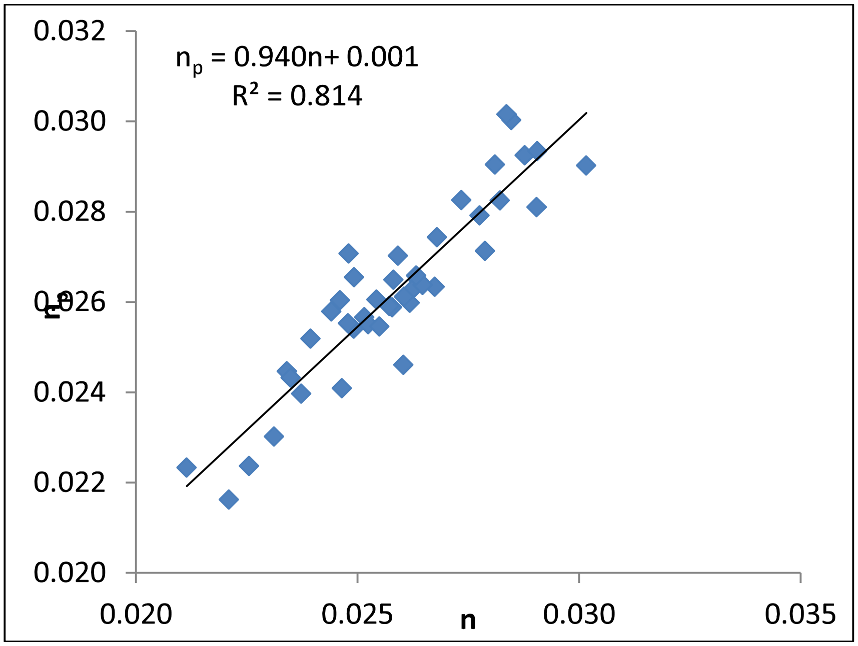

Figure 1: Relationship between Manning and the Predicted Equation

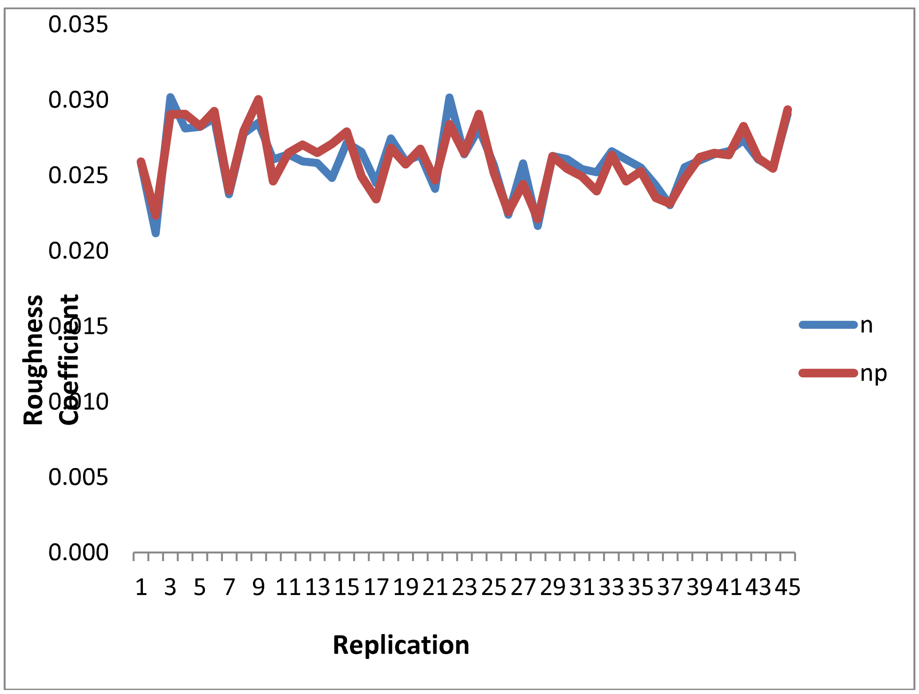

Figure 2: Manning's and predicted Roughness Coefficient

4. CONCLUSIONS

A mathematical model was developed using dimensional analysis based on the Buckingham's π theorem and a functional relationship between some channel and flow parameters was developed. The model was validated using Manning's equation. The validation of the predicted equation gave a very high correlation of 0.90 with a Coefficient of Determination, R^2, of 0.814. It also revealed that there was a good agreement between the predicted equation and Manning's equation. Thus, the developed model in this study estimates the roughness coefficient with acceptable accuracy.

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